

## Největší společný dělitel mnohočlenů v $\mathbb{Z}[x]$

**PŘÍKLAD 1:** Určete NSD polynomů  $3x^3 - 7x^2 - 4$ ,  $3x^4 + 6x^2 - 3x + 6$  v  $\mathbb{Z}[x]$ .

```
[ > restart;
> f:=3*x^3-7*x^2+7*x-4; g:=3*x^4+6*x^2-3*x+6;
      f:=3x3-7x2+7x-4
      g:=3x4+6x2-3x+6
> factor(f); factor(g);
      (3x-4)(x2-x+1)
      3(x2-x+1)(x2+x+2)
> q1:=quo(g,f,x); r1:=rem(g,f,x);
      q1:=x+7/3
      r1:=46/3+46/3x2-46/3x
> q2:=quo(f,r1,x); r2:=rem(f,r1,x);
      q2:=9x/46-6/23
      r2:=0
```

Řešení:

```
[ > NSD:=3/46*r1;
      NSD:=x2-x+1
```

**PŘÍKLAD 2:** Určete NSD polynomů  $6x^4 - 6x^3 + 3x - 3$ ,  $3x^4 - 12x^3 + 9x^2 + 3x - 3$  v  $\mathbb{Z}[x]$ .

```
[ > restart;
> f:=6*x^4-6*x^3+3*x-3; g:=3*x^4-12*x^3+9*x^2+3*x-3;
      f:=6x4-6x3+3x-3
      g:=3x4-12x3+9x2+3x-3
> factor(f); factor(g);
      3(x-1)(2x3+1)
      3(x-1)(x3-3x2+1)
> q1:=quo(f,g,x); r1:=rem(f,g,x);
      q1:=2
      r1:=3+18x3-3x-18x2
> q2:=quo(g,r1,x); r2:=rem(g,r1,x);
```

$$q2 := \frac{x}{6} - \frac{1}{2}$$

$$r2 := -\frac{3}{2} + \frac{1}{2}x^2 + x$$

```
> q3:=quo(r1,r2,x); r3:=rem(r1,r2,x);
```

$$q3 := 36x - 108$$

$$r3 := -159 + 159x$$

```
> q4:=quo(r2,r3,x); r4:=rem(r2,r3,x);
```

$$q4 := \frac{x}{318} + \frac{1}{106}$$

$$r4 := 0$$

Řešení:

```
> NSD:=3*1/159*r3;
```

$$NSD := -3 + 3x$$

**PŘÍKLAD 3:** Určete NSD polynomů  $36x^3 + 6x^2 + 150$ ,  $40x^3 + 10x + 150$  v  $\mathbb{Z}[x]$ .

```
> restart;
```

```
> f:=36*x^3+6*x^2+150; g:=40*x^3+10*x+150;
```

$$f := 36x^3 + 6x^2 + 150$$

$$g := 40x^3 + 10x + 150$$

```
> gcd(f,g);
```

$$10 + 4x^2 - 6x$$

```
> factor(f); factor(g);
```

$$6(3x + 5)(2x^2 - 3x + 5)$$

$$10(2x + 3)(2x^2 - 3x + 5)$$

```
> q1:=quo(f,g,x); r1:=rem(f,g,x);
```

$$q1 := \frac{9}{10}$$

$$r1 := 15 + 6x^2 - 9x$$

```
> q2:=quo(g,r1,x); r2:=rem(g,r1,x);
```

$$q2 := \frac{20x}{3} + 10$$

$$r2 := 0$$

Řešení:

```
> NSD:=2*1/3*r1;
```

$$NSD := 10 + 4x^2 - 6x$$

**PŘÍKLAD 4:** Určete NSD polynomů  $x^3 - 3x^2 + 5x - 3$ ,  $4x^4 + 4x^3 - 4x^2 + 20x + 24$  v  $\mathbb{Z}[x]$ .

```
[ > restart;
> f:=x^3-3*x^2+5*x-3; g:=4*x^4+4*x^3-4*x^2+20*x+24;
      f:=x3-3x2+5x-3
      g:=4x4+4x3-4x2+20x+24
> gcd(f,g);
      x2-2x+3
> factor(f); factor(g);
      (x-1)(x2-2x+3)
      4(x+2)(x+1)(x2-2x+3)
> q1:=quo(g,f,x); r1:=rem(g,f,x);
      q1:=4x+16
      r1:=72+24x2-48x
> q2:=quo(f,r1,x); r2:=rem(f,r1,x);
      q2:= $\frac{x}{24} - \frac{1}{24}$ 
      r2:=0
```

Řešení:

```
[ > NSD:=1/24*r1;
      NSD:=x2-2x+3
```

```
[ >
```